On the martingale property of certain local martingales

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The stochastic exponential $Z_t = \exp\{M_t - M_0 - (1/2)\langle M, M\rangle_t\}$ of a continuous local martingale M is itself a continuous local martingale. We give a necessary and sufficient condition for the process Z to be a true martingale in the case where $M_t = \int_0^t b(Y_u)\, dW_u$ and Y is a one-dimensional diffusion driven by the Brownian motion W. Furthermore, we provide a necessary and sufficient condition for Z to be a uniformly integrable martingale in the same setting. These conditions are deterministic and expressed only in terms of the function b and the drift and diffusion coefficients of Y. We discuss several applications, among which a deterministic criterion for absence of bubbles in a one-dimensional setting.